Project systems theory

Resit exam 2016–2017, Wednesday 12 April 2017, 14:00 – 17:00

Problem 1

(5 + 8 + 2 = 15 points)

Consider the model of a tank in which two fluids are mixed, given as

$$\dot{h}(t) = \frac{q_C(t) + q_H(t) - c\sqrt{h(t)}}{A},$$
$$\dot{T}(t) = \frac{q_C(t)\big(T_C - T(t)\big) + q_H(t)\big(T_H - T(t)\big)}{Ah(t)}.$$

Here, h is the height of the fluid level of the tank and T is the temperature. The constants $c > 0$ and $A > 0$ represent the geometry of the tank, whereas T_C and T_H are the constant temperatures of the inflowing (cold and hot) fluids, with $0 < T_C < T_H$. Finally, q_C and q_H model the inflow and are regarded as control parameters. Thus, take

$$x(t) = \begin{bmatrix} h(t) \\ T(t) \end{bmatrix}, \qquad u(t) = \begin{bmatrix} q_C(t) \\ q_H(t) \end{bmatrix},$$

as the state and input, respectively.

(a) Show that, for any desired equilibrium $h(t) = \bar{h}$, $T(t) = \bar{T}$ satisfying

$$\bar{h} > 0, \qquad T_C \leq \bar{T} \leq T_H,$$

there exists a unique constant input $q_C(t) = \bar{q}_C$, $q_H(t) = \bar{q}_H$ that achieves this equilibrium.

(b) Linearize the tank model around the equilibrium point $h(t) = \bar{h}$, $T(t) = \bar{T}$ and corresponding input $q_C(t) = \bar{q}_C$, $q_H(t) = \bar{q}_H$, as obtained in (a).

(c) Is the linearized system (internally) stable?

Problem 2

(20 points)

Consider the family of polynomials

$$\mathcal{P}(\lambda) = \big\{\lambda^3 + \theta_2\lambda^2 + a\lambda + \theta_0 \;\big|\; a \leq \theta_2 \leq 3a,\, 2a \leq \theta_0 \leq 4a\big\},$$

with a a real number. Determine for which values of a the family of polynomials $\mathcal{P}(\lambda)$ is stable (recall that a family $\mathcal{P}(\lambda)$ is stable if each polynomial belonging to $\mathcal{P}(\lambda)$ is stable).

Problem 3 (5 + 12 + 8 = 25 points)

Consider the system

$$\dot{x} = \begin{bmatrix} -1 & 2 & 0 \\ -2 & -1 & 1 \\ 0 & 0 & 2 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u.$$

(a) Is the system controllable?

(b) Find a nonsingular matrix T and real numbers α_1, α_2, α_3 such that

$$T^{-1}AT = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \alpha_1 & \alpha_2 & \alpha_3 \end{bmatrix}, \qquad T^{-1}B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

(c) Use the matrix T from problem (b) to obtain a state feedback of the form $u = Fx$ such that the closed-loop system matrix $A + BF$ has eigenvalues at -1, -1, and -2.

Problem 4 (4 + 4 + 4 + 4 + 4 = 20 points)

Consider the system

$$\dot{x} = \begin{bmatrix} 6 & 1 & 0 \\ 0 & -3 & 0 \\ -8 & -1 & -2 \end{bmatrix} x + \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} u, \qquad y = \begin{bmatrix} 1 & 0 & 1 \end{bmatrix} x.$$

Answer the following questions and explain your answers:

(a) Is the system (internally) stable?

(b) Is the system stabilizable?

(c) Is the system observable?

(d) Is the system detectable?

(e) Determine the unobservable subspace.

Problem 5 (10 points)

Prove that the characteristic equation for the matrix

$$M = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & \ddots & & 0 \\ \vdots & & \ddots & \ddots & \ddots & \vdots \\ 0 & & & \ddots & 1 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 1 \\ -m_n & -m_{n-1} & -m_{n-2} & \cdots & -m_2 & -m_1 \end{bmatrix}$$

is given as

$$\Delta_M(\lambda) = \lambda^n + m_1\lambda^{n-1} + \ldots + m_{n-1}\lambda + m_n.$$

(10 points free)